

R16

Code No: 133BD

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech II Year I Semester Examinations, February - 2024

MATHEMATICS - IV

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, MMT, AE, MIE, MSNT)

Time: 3 Hours

Max. Marks: 75

Note: i) Question paper consists of Part A, Part B.

ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.

iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART - A

(25 Marks)

- 1.a) Define Analytic function. [2]
- b) If $w = \log z$, find $\frac{dw}{dz}$ and determine where w is non-analytic. [3]
- c) Find the zeros of the function $\sin\left(\frac{1}{z}\right)$. [2]
- d) Show that the function e^z has an essential singularity at $z = \infty$. [3]
- e) Define conformal mapping. [2]
- f) Find the fixed points of the transformation $w = \frac{z-1+i}{z+2}$. [3]
- g) State dirichlet conditions. [2]
- h) State Fourier integral in complex form. [3]
- i) In heat flow, what is meant by transient state. [2]
- j) Classify the second order partial differential equations. [3]

PART - B

(50 Marks)

- 2.a) Show that $f(z) = xy + iy$ is everywhere continuous but it is not analytic.
- b) If $f(z) = u + iv$ is an analytic function, show that $u(x,y) = C$ and $v(x,y) = K$ represent two families of orthogonal curves. [5+5]

OR

- 3.a) Find the conjugate harmonic function of the harmonic function $u = a(1 + \cos\theta)$.
- b) Show that xy^2 cannot be real part of an analytic function. [5+5]

- 4.a) Evaluate $\int_c (y - x - 3x^2i) dz$, where c consists of the line segments from $z = 0$ to $z = i$ and the other from $z = i$ to $z = 1+i$.

- b) Expand the function $f(z) = \frac{z}{(z-1)(z-3)}$ as Taylor's series about $0 < |z-1| < 2$. [5+5]

OR

5. Show that using residue theorem, $\int_0^{2\pi} \frac{1}{a+b\sin\theta} d\theta = \frac{2\pi}{\sqrt{a^2-b^2}}$, $a > b > 0$. [10]

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6. Evaluate $\int_0^{2\pi} \frac{\sin^2 \theta}{a+b\cos\theta} d\theta$; ($a>b>0$). [10]

OR

QA

7. Find the bilinear transform which maps the points $z = 0, -i, -1$ into the points $w = i, 1, 0$. Find the image of the line $y = mx$ under this transformation. [10]

8. Find Fourier sine transform of $f(x) = \frac{1}{x(x^2+a^2)}$ and hence deduce cosine transform of $\frac{1}{x^2+a^2}$. [10]

OR

QA

9. If $f(x) = \begin{cases} x, & 0 < x \leq \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$ then Show that

a) $f(x) = \frac{4}{\pi} \left[\sin x - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} - \dots \right]$

b) $f(x) = \frac{\pi}{4} - \frac{2}{\pi} \left[\frac{\cos 2x}{1^2} + \frac{\cos 6x}{3^2} + \frac{\cos 10x}{5^2} + \dots \right]$ [5+5]

10. The ends A and B of a rod 20 cm long have the temperatures at 30°C and 80°C until steady-state prevails. The temperatures of the ends are changed to 40°C and 60°C respectively. Find the temperature distribution in the rod at time t. [10]

OR

11. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^2(\pi x/l)$. If it is released from rest from this position, find the displacement $y(x, t)$. [10]

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